V. *A Letter from Mr.* Colin Mac Laurin, Math. Prof. Edinburg. F. R. S. *to Mr.* John Machin, Aſtr. Prof. Greſh. & Secr. R. S. *concerning the* Deſcription of Curve Lines. *Communicated to the Royal Society on* December 21, 1732.

I Am informed that ſome Papers have been preſented to the Royal Society of late, concerning the Deſcription of Curves, in a manner that has a near affinity to that which I communicated to them of old, and have carried farther ſince; and that it would not be unſeaſonable, nor unacceptable, if I ſhould ſend an Account of what I have done further on that Subject ſince the Year 1719. The Author of thoſe Papers taught Mathematicks here privately for ſome Years, and ſometime ago (*viz.* in 1727.) mentioned to me ſome Theorems he had on that Subject; which, at the ſame time, I ſhewed him in my Papers. Some time before that, he ſhewed me a Theorem which coincided with one of thoſe in my Book, tho' he ſeemed not to have obſerved that Coincidence; and indeed Methods of that kind, are often found coincident that do not appear ſuch at firſt ſight. I am unwilling to be the Occaſion of diſcouraging any thing that is truly ingenious, and renounce any Pretenſions of appropriating Subjects to my ſelf; but, on the contrary, wiſh Juſtice may be done to every Perſon, or to any Performance in Proportion to it Merit; yet I find

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it is fit I ſhould take Precautions, leſt any one ſhould take it in his head afterwards to ſay, I take things from him which I may have had long before him; and therefore ſhall ſend you an Abſtract of what I have done in relation to this matter, ſince the Year 1719.

I have ſo much on this Subject by me, that I am at a loſs what to ſend; but at preſent I ſhall only give you an Abſtract of thoſe Propoſitions, which I take to be more nearly related to thoſe which this Author has offered to the Society from the Converſations I had with him. You know that in 1721, I printed ſeveral Sheets of a Supplement to my Book on the Deſcription of Curve Lines, which I have never yet publiſhed, having been engaged for the moſt part in Buſineſs of a different nature, and in Purſuits on other Subjects ſince that time. I ſhall firſt give you an Abſtract of that Supplement, as far as it was then printed, and ſhall ſubjoin to this, an Account of ſome Theorems I added to it the following Year, *viz.* in 1722. I was led into thoſe new Theorems by Mr. *Robert Sympſon*'s giving me at that time a Hint of the ingenious Paper, which has been ſince publiſhed in the Philoſophical Tranſactions. I had tried in the Year 1719, what could be done by the Rotation of Angles on more than two Poles; and had obſerved, that if the Interſections of the Legs of the Angles were carried over Right Lines, as in Sir *Iſaac Newton*'s Deſcription, the Dimenſions of the Curve were not raiſed by this Increaſe of the Number of Poles, Angles, and Right Lines; and therefore neg-

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lected this at that time, as of no use to me; confining my self to two Poles only, and varying the Motions of the Angles as you find them in my Book. I found this by inquiring in how many Points the Locus could cut a Right Line drawn in its Plane, and found, by a Method I often use in my Book, that it could meet it in two Points only.

Having found then, that three or more Poles, were of no more Service than two, while the Intersections were carried over fixed Right Lines; I thought it needless to prosecute that Matter then, since by increasing the Number of Poles, my Descriptions would become more complex without any Advantage. But in *June* or *July*, 1722, upon the Hint I got from Mr. *Sympson* of Mr. *Pappus*'s Porisms, I saw that what he has there ingeniously demonstrated, might be considered as a Case of the above-mentioned Description of a Conick Section, by the Rotation of any Number of Angles about as many Poles; the Intersections of their Legs, in the mean time, being carried over fixed Right Lines, excepting that of two of them which describes the Locus. For by substituting Right Lines in place of the Angles, in certain Situations of the Poles and of the fixed Right Lines, the Locus becomes a Right Line; as for Example, in the Case of three Poles, when these three are in one Right Line, in which Case the Locus is a Right Line, which is a Case of the Porism.

'Twas this led me to consider this Subject anew; and first I demonstrated the Locus to be a Conick Section algebraically; and found Theorems for

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drawing Tangents to it, and determining its Asymptotes. I also drew from it at that time a Method of describing a Conick Section through five given Points [a]. This encouraged me to substitute Curves for the Right Lines, to see if by this Method I could be enabled to carry on my Theorems about the Descriptions of Lines through given Points to the higher Orders of Lines. Some of the Theorems I found at that time, I now send you. In *November* 1722, looking into Sir *Isaac*'s *Principia*, I saw that the Description of the Conick Section by three Right Lines, moving as above, about three Poles, could be immediately drawn from his 20th Lemma, which itself is a Case of this Description. This gradually led me to seek Geometrical Demonstrations for the whole, as far as it related to the Conick Sections. I send you some Leaves of this Paper dated at *Nancy*, *November* 1722. Since that time, I have not added much to this Subject, but what relates to the drawing Tangents, determining the Asymptotes, and the *Puncta Duplica*, or *Multiplicia* of these Curves. I considered it the less, that I did not find it more advantageous in any respects, than the Method I had considered in my Book, or more general.

In 1727 I added to a Chapter in my Algebra, which is very publick in this Place, an Algebraick Demonstration of the Locus, when three Poles are employed; and the Method of describing a Conick

[a] *The Paper on this Subject I have, is dated* July 31, 1722, *at Sea, being then in my way to* London, *going for* Cambray.

Section

Section through five given Points, subjoining at the same time, that if more Poles are employed, and Angles or Right Lines, the Locus was still a Conick Section; which I thought was a remarkable Property of the Conick Sections not observed before.

These Things I intended to put in order, and publish in the Supplement to my Book, a Part of which has been printed since the Year 1721. I have in my view also to give several other Things in that Supplement; two of which, I shall only just mention at present, because I believe they are foreign to the present Affair. I subjoin a Problem determining the Figure of a Fluid, whose Parts are supposed to be attracted to two or more Centers; and a Solution of a general Problem about the Collision of Bodies.

The Author of the Papers given in to the Royal Society, will not refuse that I shewed him the Theorems, I now send you, in 1727. He owned it last Summer at least: I am to publish these very soon. Whether he has carried the Subject farther, I leave to the Judgment of the Gentlemen to whom they were referred. As to the Demonstrations, it would take some time to put them in a proper Form to be published. You who have so nice a Taste of Demonstrations, will easily allow, that it ought not to be done in a hurry. I could send those that are Algebraick easily; but do not care to send those that are Geometrical, till I have leisure. I could not have been called to this in a worse Season of the Year than now, when I begin my Classes, and have

few

few Minutes in the Day my own. I ought to make an Apology for this long Letter; but thought you was the Perſon of my Acquaintance moſt proper to ſend this to. I am, with great Eſteem,

S I R,

Your moſt obedient

moſt humble Servant,

Colin Mac Laurin.

An Abſtract of what has been printed ſince the Year 1721, as a Supplement to a Treatiſe concerning the Deſcription of Curve Lines publiſhed in 1719, and of what the Author propoſes to add to that Supplement.

I. IN the firſt Part of the Supplement, there is a general Demonſtration given of the Theorem, that if two Lines of the Orders or Dimenſions, expreſs'd by the Numbers m and n, be deſcribed in the ſame Plane, the greateſt Number of Points in which theſe Lines can interſect each other, will be mn, or the Product of the Numbers which expreſs the Dimenſions of the Lines, or the Orders to which they belong.

II. In the next Part, Theorems are given for drawing Tangents to all the Curves that were deſcribed in that Treatiſe by the Motions of Angles upon

upon given Lines. Their Aſymptotes are alſo determined by more ſimple Conſtructions than thoſe which are ſubjoined to their Deſcriptions in that Treatiſe. Of theſe we ſhall give one Inſtance here.

Suppoſe the invariable Angles (*Fig.* 1 and 2) FCG, KSH, to revolve about the fixed Points or

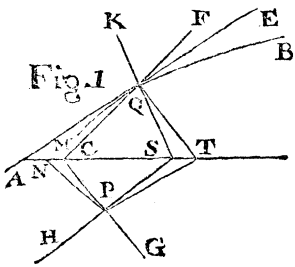


Fig. 1

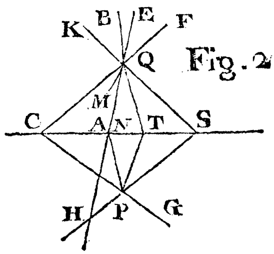


Fig. 2

Poles, C and S. Suppoſe the Interſection of the two Sides C F, S K, to be carried over the Curve B Q M, whoſe Tangent at the Point Q is ſuppoſed to be the Right Line A E; and let it be required to draw a Tangent at P to the Curve Line deſcribed by P the Interſection of the other two Sides C G and S H.

Conſtruction. Draw Q T conſtituting the Angle S Q T, equal to C Q A, on the oppoſite Side of S Q, that Q A is from C Q; and let Q T meet C S (produced if neceſſary) in T. Join P T, and conſtitute the Angle C P N equal to S P T, on the oppoſite Side of C P, that P T is from S P, and the Right Line P N ſhall be a Tangent at P, to the Curve deſcribed by the Motion of P, which is always ſuppoſed to be the Interſection of C G and S H.

The Aſymptotes of the Curve, deſcribed by P, are determined thus. Find, as in the abovementioned Treatiſe, when theſe Sides become parallel, whoſe Interſection is ſuppoſed to trace the Curve ; which always happens when the Angle C Q S becomes equal to the Supplement of the Sum of the invariable Angles F C G, K S H, to four Right ones, becauſe the Angle C P S then vaniſhes. Suppoſe (in *Fig.* 3 and 4,) that when this happens, the Interſection of the Sides C F, S K is found in Q.

Fig. 3

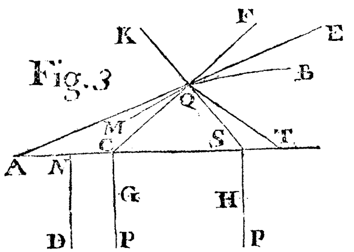
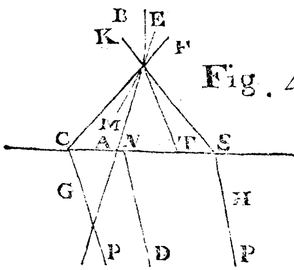


Fig. 4



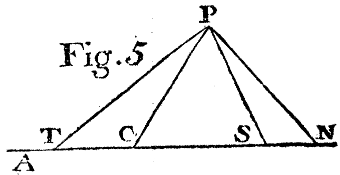
Conſtitute the Angle S Q T equal to C Q A, as before, and let Q T meet C S in T. Take C N equal to S T, the oppoſite Way from C that S T lies from S. Through N draw D N parallel to C G or S H, which are now parallel to each other, and D N ſhall be an Aſymptote of the Curve deſcribed by the Motion of P.

If in place of a Curve Line B Q M, a fixed Right Line A E be ſubſtituted, then the Point P will deſcribe a Conick Section, whoſe Tangents and Aſymptotes are determined by theſe Conſtructions. In this Supplement, it is afterwards ſhewn how to draw the Tangents and Aſympotes of all the Curves which

which are described in the above-mentioned Treatise by more Angles and Lines.

III. The same Method is afterwards applied for to draw Tangents to Lines described by other Motions than those which are considered in that Treatise; of which the following is an Instance. Suppose that the Lines C P and S P revolve about the Poles C and S, so that the Angle A C P bears always the same invariable Proportion to A S P, suppose that of *m* to *n*. In the Line C S, take

Fig. 5



the Point T, so that S T may be to C T in that same Proportion of *m* to *n*; and this Point T will be an invariable Point, since C T is to C S, as $m-n$ to *n*. Draw T P, and constitute the Angle S P N, equal to C P T, so that P N and P T, may lie contrary ways from S P and C P, and P N shall be a Tangent of the Curve described by the Motion of the Point P. Several other Theorems of this kind are subjoined here.

IV. After these, Lines or Angles are supposed to revolve about three or more Poles, and the Dimensions of the Curves with their Tangents and Asymptotes are determined. Suppose in the first Place,

 that

that the three Poles are C, S and D, and that Lines or Rulers C R, S Q, Q D R, revolve about these Poles. The Line which revolves about D, serves only to guide the Motion of the other two, so that its Intersection

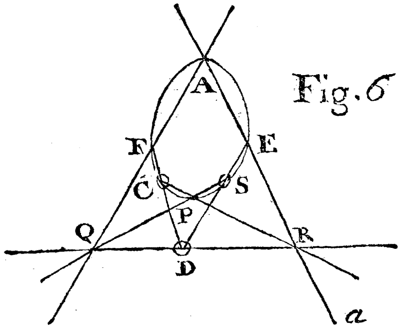


Fig. 6

with each of them being carried over a fixed Right Line, their Intersection with each other describes the Locus, which is shewn to be a Conick Section. The Intersection of Q D R with S Q, is supposed to be carried over the fixed Right Line A F; the Intersection of the same Q D R with C R, is supposed to be carried over the fixed Right Line A E; and in the mean time, the Intersection of the Right Lines S Q, C R, that revolve about the Poles S and C, describes a Conick Section.

This Conick Section passes through the Poles C and S; and if you produce D C and D S, till they meet with A Q and H R in F and E, it will also pass through F and E: It also passes always through A the Intersection of the fixed Lines Q F and E R; from which this easy Method follows for drawing a Conick Section through five given Points. Suppose that these five given Points are A, F, C, S and E: Join four of them by the Lines A F, F C, A E, E S, and produce two of these F C, E S, till they meet, and by their Intersection give the Point D. Suppose infinite Right Lines revolve about this Point

Point D, and the Points C and S, two of thoſe that were given, and let the Interſections of the Line revolving about D, with thoſe that revolve about C and S, be carried over the given Right Lines A E, C F; and the Interſection of thoſe that revolve about C and S with each other, will, in the mean Time, deſcribe a Conick Section, that ſhall paſs through the five given Points A, F, C, S and E.

It is then ſhewn, that when C, S and D are taken in the ſame Right Line, the Point P deſcribes a Right

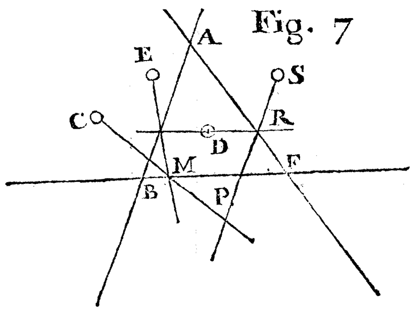


Fig. 7

Line; as alſo when C, S and A are in the ſame Right Line; which alſo follows from what is demonſtrated in that very ingenious Paper concerning *Pappus*'s Poriſms, communicated by Mr. *Sympſon*, Profeſſor of Mathematicks at *Glaſgow* publiſhed in the *Phil. Tranſ.* N°. 377.

In the next Place it is ſhewn, that if four Right Lines revolve about four Poles C, S, D and E, and thoſe that revolve about D and E, ſerve only to guide thoſe that revolve about C and S; ſo that Q and R, the Interſections of that which revolves about D, with thoſe that revolve about E and S, be carried over the fixed Lines A B and A F; and M the

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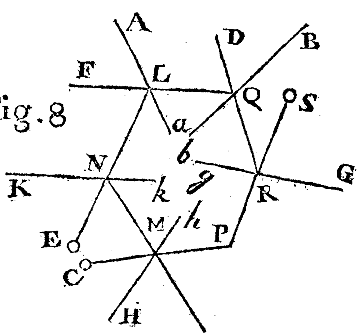
Interſection of that which revolves about E with that which revolves about C, be carried over a third fixed Line B F, then the Interſection P of thoſe that revolve about C and S, will, in the mean time, deſcribe a Conick Section, and not a Curve of a higher Order. The Conick Section degenerates into Right Lines, when C P and S P coincide at the ſame time with the Line C S, that joins the Poles C and S, as in the preceding Deſcription; which coincides again with what is demonſtrated in the abovementioned ingenious Paper.

After this it is ſhewn generally, that tho' the Poles and Lines revolving about them be increaſed to any Number, and the fixed Lines over which ſuch Interſections, as we deſcribed in the two laſt Caſes, are ſuppoſed to be carried, be equally increaſed, the Locus of the Point P will never be higher than a Conick Section: That is, let a Polygon of any number of Sides have all its Angles, one only excepted, carried over fixed Right Lines, and let each of its Sides produced, paſs through a a given Point or Pole, and that one Angle which we excepted, will either deſcribe a ſtreight Line, or Conick Section.

Fig. 8



Thus if a hexagonal Figure L Q R P M N, have all its Angles excepting P carried re-

ſpectively

ſpectively over the fixed Right Lines A *a*, B *b*, G *g*, H *h*, K *k*, the Point P in the mean time will deſcribe a Conick Section, or a Right Line. The Locus of P is a Right Line when C P and S P coincide together with the Line C S. All theſe things are demonſtrated geometrically.

V. After this, Angles are ſubſtituted in place of Right Lines revolving about theſe Poles; and it is ſtill demonſtrated geometrically, that the Locus of P is a Conick Section or Right Line.

Suppoſe that there are four Poles C, S, D and E, about which the invariable Angles P C Q, P S R, R D M, M E Q revolve; and that Q, M and R, the Interſections of the Legs C Q and E Q, of E M and D M, and of D R and S R, are carried over the fixed Right Lines A *a*, B *b*, and G *g* reſpectively, then the Locus of P is a Conick Section, when C P and S P do not coincide at once with the Line C S, but is a Right Line when C P and S P coincide at the ſame time with C S, and never a Curve of a higher Order.

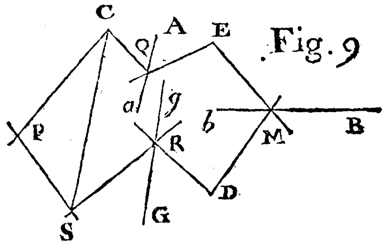


Fig. 9

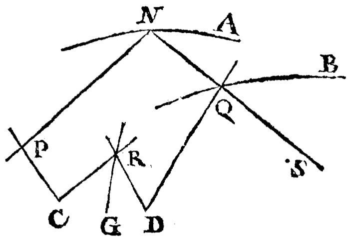
VI. Having demonſtrated this which ſeems a remarkable Property of the Conick Sections or Lines of the Second Order; I proceed to ſubſtitute Curve Lines in place of Right Lines in theſe Deſcriptions, (as I always do in the Treatiſe concerning the Deſcription

ſcription of Lines) and to determine the Dimenſions of the Locus of P, and to ſhew how to draw Tangents to it to determine its Aſymptotes, and other Properties of it. I had obſerved in 1719, that by increaſing the Number of Poles and Angles beyond two, the Dimenſions of the Locus of P, did not riſe above thoſe of the Lines of the Second Order, while the Interſections moved on Right Lines; and therefore I did not think it of uſe to me then to take more Poles than two, ſince by taking more, the Deſcriptions became more complex without any Advantage. When the Interſections are carried over Curve Lines, the Dimenſions of the Locus of P riſe higher, but the Curves deſcribed, have *Puncta Duplicia,* or *Multiplicia*, as well as when two Poles only are aſſumed; and therefore this Speculation is more curious than uſeful. However, I ſhall ſubjoin ſome of the Theorems that I found on this Subject concerning the Dimenſions of the Locus of P, and the drawing Tangents to it.

1. If in *Fig.* 6. you ſuppoſe Q and R to be carried over Curve Lines of the Dimenſions m and n reſpectively, then the Point P may deſcribe a Locus of $2mn$ Dimenſions.

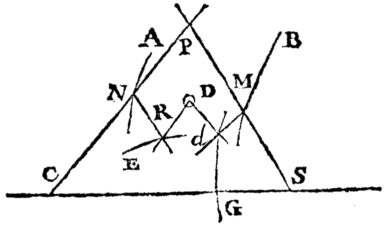
2, If in *Fig.* 8. you ſuppoſe L, Q, R, M, N, to be carried over Curve Lines of the Dimenſions m, n, r, s, t, reſpectively, the Locus of P may ariſe to $2mnrst$ Dimenſions, but no higher; and if in place of Lines revolving about the Poles, you uſe invariable Angles, the Dimenſions of the Locus of P will riſe no higher.

3. I then aſſumed three Poles C, D and S, and ſuppoſed one of the Angles S N L, to have its angular Point N carried over the Curve A N, while the Leg N Q paſſes always through S, as in the Deſcription in the Treatiſe of the General Deſcription of Curve Lines, while the Angles Q D R, R C P, revolve about the Poles D and C: I ſuppoſe alſo the Interſections Q and R to be carried over the Curve Lines B Q, G R, and that the Dimenſions of the curve Lines A N, B Q, G R, are *m*, *n*, *r*, reſpectively; and find that the Locus of P may be of $3mnr$ Dimenſions; but that the Point C is ſuch, that the Curve paſſes through it as often as there are Units in $2nmr$.



4. If any number of Poles are aſſumed, ſo as to have Angles revolving about them, as about C and D in the laſt Article, and the Interſections are carried over other Curves, the Dimenſions of the Locus of P will be equal to the triple Product of the Number of Dimenſions of all the Curves employed in the Deſcription.

5. If the invariable Angles P N R, P M Q, move ſo that while the Sides P N, P M, paſs always through the Poles C and S, the angular



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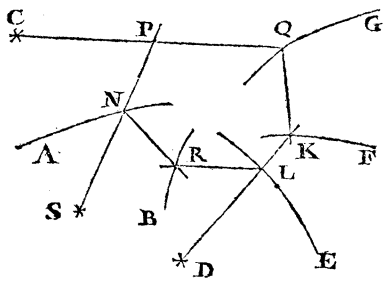
Points N and M deſcribe the Curves A N and B M; and at the ſame time, the invariable Angle R D Q, revolve about the third Pole D, ſo that the Interſections R and Q deſcribe the Curves E R and G Q; then the Dimenſions of the Locus of P, when higheſt, ſhall be equal to the quadruple Product of the Numbers that expreſs the Dimenſions of the given Curves A N, E R, G Q and B M, multiplied continually into each other. If more Poles are aſſumed, about which Angles be ſuppoſed to move, as R D Q moves about D in this Deſcription, and the Interſections of the Sides be ſtill carried over Curves, as in this Example; the Dimenſions, of the Locus of P, when higheſt, ſhall ſtill be found equal to the quadruple Product of all the Numbers that expreſs the Dimenſions of the Curves employed in this Deſcription.



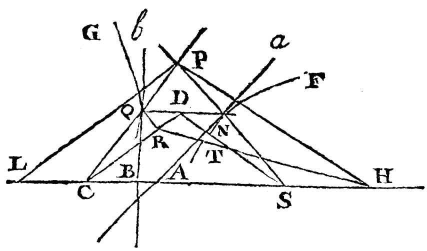
6. Suppoſe that the three invariable Angles P Q K, K L R, R N P, move over the Curves G Q, E L, A N, ſo that the Sides P Q, K L, P N produced, paſs always through the Poles C, D, S, and that the Interſections of their Sides K and R, at the ſame time move over the Curves F K and B R; and

and the Dimenſions of the Locus of P when higheſt, ſhall be equal to the Product of the Numbers that expreſs the Dimenſions of the given Curves multiplied by Six. If more Poles, with the neceſſary Angles and Curves, are aſſumed betwixt C and D, as here D is aſſumed betwixt C and S, and the Motions be in other reſpects like to what they are in this Example; then in order to find the Dimenſions of the Locus of P when higheſt, raiſe the Number 2 to a Power whoſe Index is leſs than the Number of Poles by a Unit; add 2 to this Power, and multiply the Sum by the Product of the Numbers that expreſs the Dimenſions of the Curves employed in the Deſcription; and this laſt Product ſhall ſhew the Dimenſions of the Locus of P when higheſt.

I am able to continue theſe Theorems much farther: But it is not worth while, eſpecially ſince I find that there is not any conſiderable Advantage obtained by increaſing the number of Poles above the Method delivered in the abovementioned Treatiſe of the Deſcription of Curve Lines. On the contrary, the Deſcriptions there given by means of two Poles, will produce a Locus of higher Dimenſions by the ſame number of Curves and Angles, than theſe that require three or more Poles; and are therefore preferable, unleſs perhaps in ſome particular Caſes.

VII. However, I have alſo found how to draw Tangents to the Curves that ariſe in all theſe Deſcriptions; of which I ſhall give one Inſtance where three Right Lines are ſuppoſed to revolve about three Poles, and two of their Interſections are ſuppoſed to be carried over given Curve Lines, and the third describes the Locus required.

Let the Right Lines C Q, S N, D N, revolve about the Poles C, S, D, where that which revolves about D, serves to guide the Motion of the other two; its Intersection with C Q moving over the Curve G Q, while its Intersection with S N moves over the Curve F N. Suppose that the Right Line B *b*



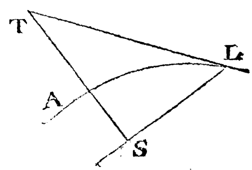
touches the Curve G Q in Q, and that the Right Line A *a* touches the Curve F N in N. In order to draw a Tangent to the Locus of P; join D C, D S and C S, and constitute the Angle D Q R, equal to C Q B, so that Q R lie the contrary way from Q D that Q B lies from Q C, and let Q R meet D C in R. Constitute also the Angle D N T, equal to S N A with the like precaution, and let N T meet D S in T. Join R T, and produce it till it meet C S in H; then join P H, and make the Angle C P L equal to S P H, so that P L and P H, may lie contrary ways from C P and S P; and P L shall be a Tangent at P, to the Locus described by P, the Intersection of C Q and S N.

I have

I have alſo applied this Doctrine to the Deſcription of Lines through given Points. But I ſuppoſe I have ſaid enough at preſent on this Subject; and ſhall conclude, after obſerving that in the abovementioned Treatiſe, I have given an eaſy Theorem for calculating the Reſiſtance of the Medium when a given Curve is deſcribed with a given centripetal Force in a reſiſting Medium, which I ſhall here repeat, becauſe it has been miſrepreſented in a foreign Journal.

Let V expreſs the centripetal Force with which the Body that is ſuppoſed to deſcribe the Curve, is acted on in the Medium; let v expreſs the centripetal Force with which the ſame Curve could be deſcribed in a Void; ſuppoſe $z = \frac{V}{v}$, and the Reſiſtance ſhall be proportional to the Fluxion of z multiplied by the Fluxion of the Curve, ſuppoſing the Area deſcribed by a Ray, drawn from the Body to the Center of the Forces, to flow uniformly. Let this Theorem be compared with what the celebrated Mathematician mentioned by that Journaliſt has given on the ſame Subject, and it will eaſily appear what judgment is to be made of his Aſſertion; and ſince ſeveral Perſons, and particularly the Gentleman mentioned above in this Paper, teſtify that I communicated to them this Theorem before any Thing was publiſhed on this Subject by the learned Mathematician he names, his Obſervation on this Occaſion muſt appear the more groundleſs.

From this Theorem, I draw this very general Corollary, that if the Curve is ſuch as could be deſcribed in a Void by a centripetal Force, varying according to any Power of the Diſtance, then the Denſity of the Medium in any place, is reciprocally proportional to the Tangent of the Curve at that place, bounded at one Extremity by the Point of Contact, and, at the other, by its Interſection with a Perpendicular raiſed at the Center of the Forces to the Ray drawn from that Center to the Point of Contact. Let A L be the Curve deſcribed by a Force directed to the Point S; let L T touch the Curve at L, and raiſe S T perpendicular to S L, meeting L T in T, and the Denſity in L ſhall be univerſely as L T, if the Reſiſtance be ſuppoſed to obſerve the compound Proportion of the Denſity, and of the Square of the Velocity.



Beſides what I have obſerved here, I propoſe to illuſtrate and improve ſeveral other Parts of the Treatiſe concerning the Deſcription of Curve Lines in this Supplement.

That Treatiſe requires theſe Additions and Illuſtrations the more, that tho' the whole almoſt was new, it was publiſhed in a hurry, when I was very young, before I had time to conſider ſufficiently which were the beſt ways of demonſtrating the Theorems, or reſolving the Problems, for which this Supplement I hope, will make ſome Apology.

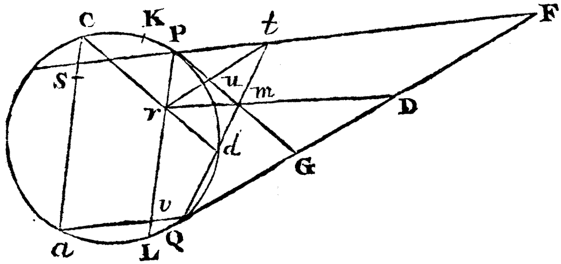
N. B. *The following Paper, dated at* Nancy, Novem. 27, 1722, *is that which the Author mentions in his Letter.*

SECTIO

SECTIO I.

Prop. 1. *Quæ respicit descriptionem Linearum* 1.

Circa Polos C, B, D, moveantur rectæ C *d*, B *m*, D *r*, & ducatur concursus crurum B *m*, D *r* per rectam



datam P G, concursus crurum C *d*, D *r* per rectam PQ etiam datam & concursus crurum C *d*, B *d* sectionem conicam describet.

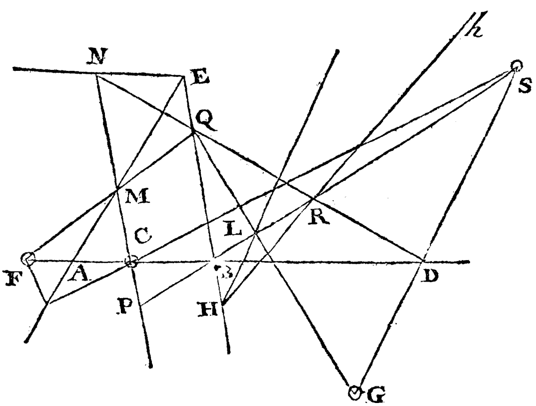
Ducatur *r t* parallela rectæ B D positione datæ occurrens rectæ B *d* in *t*; jungatur P *t* & producatur donec occurrat rectæ B D in F; atque dabitur punctum F. Quippe cum detur ratio *r u* ad *r t*, eadem enim est ac D G ad D B, ob similes figuras D *m* B G & *r m t a* sitq; *r u* ad *r t*, ut Q G ad Q F, dabitur etiam ratio Q F ad Q G; adeoque ob datam Q G dabitur Q F, & proinde punctum F & recta P F. Cum igitur B *t* & C *r* partes abscindant P *t*, P *r*, à rectis positione datis P F, P Q, in data semper ratione erit illarum concursus *d* in sectione conica per Lemma 20. Lib. 1. Princip. D. Neutoni.

Si

Si punctum D sumatur ubicunque in recta B F, sitque semper D G ad Q G ut B D ad Q F, eadem erit sectio conica quam *d* describet.

Sectio conica transit per C, P, B & *a* complendo parallelogrammum, P S *a* Q. Transit etiam per L ubi recta B G producta occurrit ipsi P *q*, ut etiam per K ubi recta C D secat datam P G. Unde pentagonum P K C L B sectioni inscribitur. Et si detur quinque puncta C, K, P, B, L, per quæ ducenda est sectio conica vel circa pentagonum datum C L B P K circumscribenda sit sectio conica producantur duo quævis latera C K, L B ad occursum suum D dein jungantur reliqua P L, P K & ducantur semper concursus rectarum C *d*, D *r*; B *d*, D R per rectas illas P L, P R, & concursus *d* sectionem describet.

P R O P. II.



Circa data puncta F, C, G S, ut Polos moveantur, rectæ F Q, C N, G Q, S L & concursus rectarum F Q

FQ & CN, FQ & GQ, GQ & SL, puncta scil. M, Q, L, semper contingant rectas positione datas AE, BE, HL; & concursus rectarum CN, SL describet sectionem conicam.

Occurrant rectæ AM, HR ipsi BQ in E & H. Jungantur CF & GS quæ sibi mutuo occurrunt in D, jungatur DQ quæ occurrat rectis CM, SL in N & R; & si jungantur EN, & HR, erunt EN & HR, rectæ positione datæ per Lemma I*. Quippe cum sint puncta F, C, D, in eadem recta Linea & concursus rectarum FM, CM & FQ, DQ percurrant datas rectas, concursus crurum CM, DQ, etiam continget datam. Et simili ratione cum sint S, D, G in eadem recta concursus rectarum DQ, SL etiam continget datam.

Omissis igitur Polis F & G, invenienda est curva quam concursus rectarum CN, SL; viz. P describet dum rectis CN, DN, SR revolventibus circa Polos C, D, S concursus rectarum CN, DN contingit datam EN & concursus rectarum SR, DN contingit datam HR, eam vero sectionem esse conicam ex Prop. præcedenti est manifestum.

N. B. *The Papers referred to were published in a little Treatise entituled,* Exercitatio Geometrica de descriptione Curvarum. *Londin.* 1733. 4*to*.